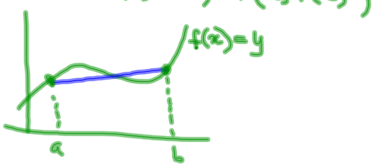
Average Rate of Change

Definition: ARC of f on $[a,b]$ is

$$\frac{f(b)-f(a)}{b-a}$$

Geometric Interpretation: slope of secant line between $(a, f(a))$ & $(b, f(b))$



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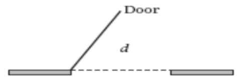
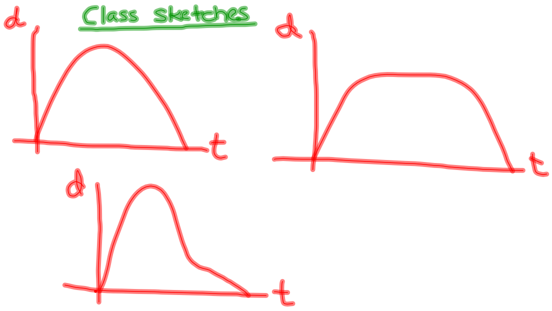


The diagram shows a door with an automatic closer. At time $t = 0$ s, someone pushes the door. It swings open, slows down, stops, starts closing, then slams shut at time $t = 7$ s. As the door is in motion, the number of degrees, d, it is from its closed position depends on t.

1. Sketch a reasonable graph of d versus t.

Class sketches



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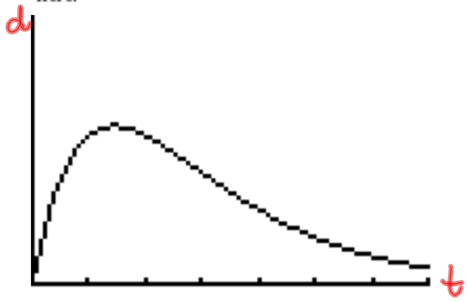
2. Suppose that d is given by the equation

$$d = 200t \cdot 2^{-t}$$

Plot this graph on your grapher. Sketch the results here.



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3. Make a table of values of d for each second from $t = 0$ through $t = 10$. Round to the nearest 0.1°.

t	d
0	0
1	100
2	100
3	75
4	50
5	31.3
6	18.8
7	10.9
8	6.3
9	3.5
10	2.0

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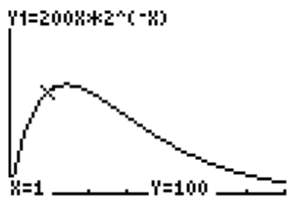
4. At time $t = 1$ s, does the door appear to be opening or closing? How do you tell?



the degree measure d is increasing when t=1 so the door appears to be opening.

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5. What is the average rate at which the door is moving for the time interval [1, 1.1]? Based on your answer, does the door seem to be opening or closing at time $t = 1$? Explain.

$$ARC = \frac{d(1.1)-d(1)}{.1} = 26.336°/\text{sec}$$

Since the average rate of change is positive, d is increasing on the interval [1, 1.1] so the door seems to be opening at t=1.

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6. By finding average rates using the time intervals [1, 1.01], [1, 1.001], and so on, make a conjecture about the *instantaneous* rate at which the door is moving at time $t = 1$ s.

$$\frac{d(1.01) - d(1)}{.01} = 30.234$$

$$\frac{d(1.001) - d(1)}{.001} = 30.640$$

$$\frac{d(1.0001) - d(1)}{.0001} = 30.681$$

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7. In calculus you will learn by four methods:
- algebraically,
- numerically,
- graphically,
- verbally (talking and writing).

What did you learn as a result of doing this Exploration that you did not know before?

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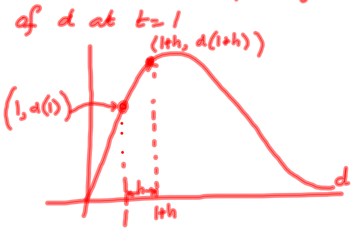
Instantaneous Rate of Change of d at t=1



$$IRC = \lim_{h \to 0} \frac{d(1+h) - d(1)}{h}$$

$$= \lim_{h \to 0} \frac{200(1+h) \cdot 2^{-(1+h)} - 200 \cdot 2^{-1}}{h}$$

$$= \lim_{h \to 0} \left[\frac{\frac{200(1+h)}{2^{(1+h)}} - 100}{h} \right]$$

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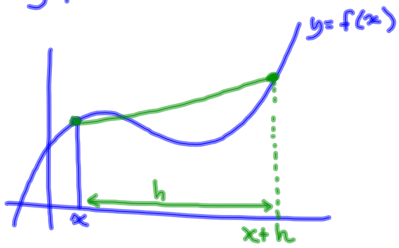
Instantaneous Rate of Change of f at any point x.



$$ARC = \frac{f(x+h) - f(x)}{h}$$

$$IRC = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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If $f(x) = 3x^2 + x$, find the IRC of f when $x = 2$.

$$IRC = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{3(2+h)^2 + (2+h) - 14}{h}$$

$$= \lim_{h \to 0} \frac{3(\cancel{4} + 4h + h^2) - \cancel{12} + h}{h}$$

$$= \lim_{h \to 0} \left(\frac{13h + 3h^2}{h} \right)$$

$$= \lim_{h \to 0} (13 + 3h)$$

$$= 13$$

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